Quadratic Relations Review

1) State whether each relation is quadratic. Justify your decision.

a. $y = 4x - 5$ No, first degree exponent, or linear

b.

X	Y
-3	56
-2	35
-1	18
0	5
1	-4
2	-9
3	-10

1st: $35-56 = -21$; $18-35 = -17$; $5-18 = -13$; $-4-5 = -9$; $-9-(-4) = -5$; $-10-(-9) = -1$

2nd: $-17-(-21) = 4$; $-13-(-17) = 4$; $-9-(-13) = 4$; $-5-(-9) = 4$; $-1-(-5) = 4$

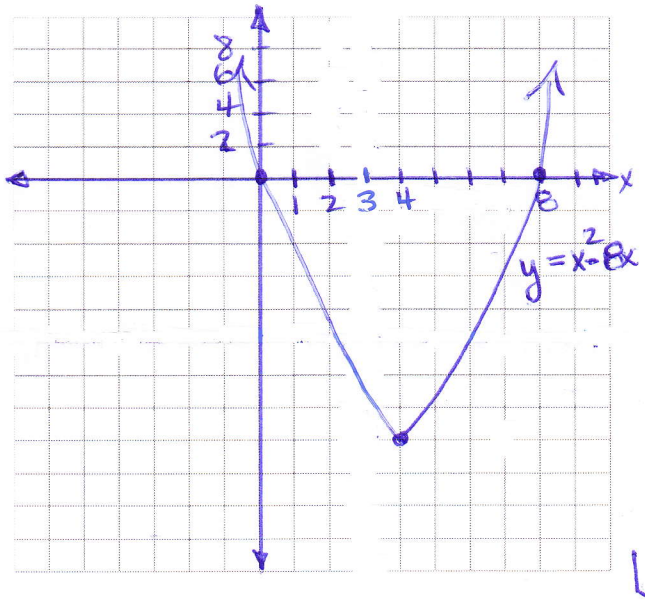
2nd differences are constant. Quadratic

c. $y = 2x(x-5)$ yes, second degree exponent

2) Graph each quadratic relation and determine

a. The equation of the axis of symmetry
b. The coordinates of the vertex
c. The y-intercept
d. The zeros

i. $Y = x^2 - 8x$



$y = x(x-8)$

zeroes: 0 & 8

axis of symmetry $= \frac{0+8}{2}$

$x = 4$

Vertex $(4, -16)$

y-intercept $= 0$

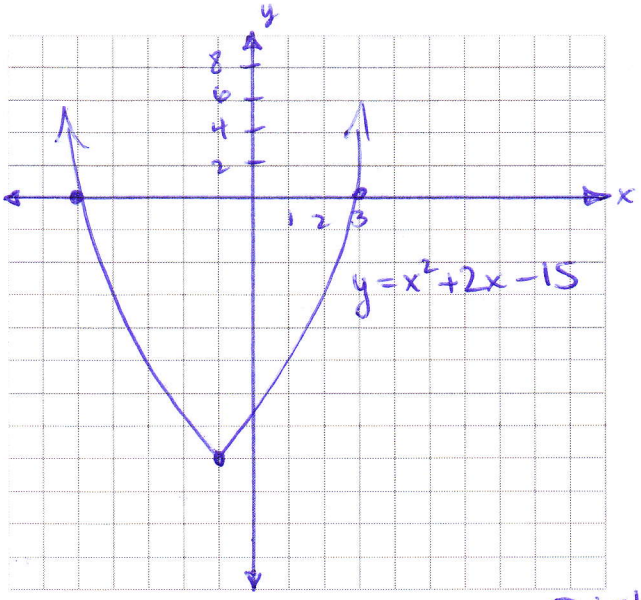
sub $x = 4$

$y = 4^2 - 8(4)$

$= 16 - 32$

$= -16$

ii) $y = x^2 + 2x - 15$



$y = x^2 + 2x - 15$

$= (x+5)(x-3)$

P: -15
S: +5 -3 = +2
I: +5 & -3

zeroes: -5 & $+3$

axis of symmetry $= \frac{-5+3}{2}$

$x = -1$

sub $x = -1$

$= (-1)^2 + 2(-1) - 15$

$= 1 - 2 - 15$

$= -16$

Vertex $(-1, -16)$

y-intercept $= -15$

3) The roots of a quadratic relation are -2 and 5, and the second differences are negative.

a. Is the y-value of the vertex a maximum value or a minimum value? Explain.

maximum because the 2nd differences are negative

b. Is the y-value of the vertex positive or negative? Explain

Positive because its between the two zeroes and it opens down.

c. Calculate the x-coordinate of the vertex

$$x = \frac{-2+5}{2}$$

$$x = 1.5$$

4) What does *a* in the equation $y = ax^2+bx+c$ tell you about the parabola?

- positive a means it opens up
- negative a means it opens down.

5) The Rudy Snow Company makes custom snowboards. The company's profit can be modelled with the relation $y = -6x^2+42x-60$, where x is the number of snowboards sold (in thousands) and y is the profit (in hundreds of thousands of dollars).

a. How many snowboards does the company need to sell to break even?

$$= -6x^2+42x-60$$
$$= -6(x^2-7x+10)$$
$$= -6(x-5)(x-2)$$

∴ they need to sell 5000 or 2000 to break even.

b. How many snowboards does the company need to sell to maximize their profits?

$$x = \frac{5+2}{2}$$

$$x = 3.5$$

∴ they need to sell 3500 to maximize their profits.

6) The x-intercepts of a parabola are -2 and 7, and the y-intercept is -28.

a. Determine the coordinates of the vertex.

b. Determine an equation for the parabola.

$y = a(x+2)(x-7)$

Sub (0, -28) →

$-28 = a(0+2)(0-7)$

$-28 = a(2)(-7)$

$\frac{-28}{-14} = \frac{-14a}{-14}$

$a = 2$

$y = 2(x+2)(x-7)$

7) Determine an equation for each parabola.

a. The x-intercepts are 5 and 9, and the y-coordinate of the vertex is -2.

axis of symmetry $= \frac{5+9}{2}$

$x = 7$

$y = 0.5(x-5)(x-9)$

Vertex $(7,-2)$ ↗ sub in

$y = a(x-5)(x-9)$

$-2 = a(7-5)(7-9)$

$-2 = a(2)(-2)$

$\frac{-2}{-4} = \frac{-4a}{-4}$

$a = 0.5$

b. The x-intercepts are -3 and 3, and the parabola passes through the point (2,20).

$y = a(x+3)(x-3)$

$20 = a(2+3)(2-3)$

$20 = a(5)(-1)$

$\frac{20}{-5} = \frac{-5a}{-5}$

$a = -4$

$y = -4(x+3)(x-3)$

8) A bus company usually transports 12 000 people per day at a ticket price of $1. The company wants to raise the ticket price. For every $0.10 increase in the ticket price, the number of riders per day is expected to decrease by 400. Calculate the ticket price that will maximize revenue.

$= (12000 - 400x)(1 + 0.10x)$

$= 12000 + 1200x - 400x - 40x^2$

$= -40x^2 + 800x + 1200$

$= -40(x^2 - 20x - 300)$

$= -40(x+10)(x-30)$

$x+10=0$ OR $x-30=0$

$x = -10$ $x = 30$

axis of symmetry $= \frac{-10+30}{2}$

$x = 10$

ticket price

$(1 + 0.10x)$

$(1 + 0.10(10))$

$(1 + 1)$

(2)

∴ a ticket price of $2 will maximize revenue.

9) Expand and simplify

a. (X+5)(X+4)

$= x^2+4x+5x+20$

$= x^2+9x+20$

b. (2x-3)(2x+3)

$= 4x^2+6x-6x-9$

$= 4x^2-9$

c. (4x-2y)(5x+3y)

$= 20x^2+12xy-10xy-6y^2$

$= 20x^2+2xy-6y^2$

d. $(2x+6)^2$

$= (2x+6)(2x+6)$

$= 4x^2+12x+12x+36$

$= 4x^2+24x+36$

e. 2x(4x-y)(4x+y)

$= 2x(16x^2+4xy-4xy-y^2)$

$= 2x(16x^2-y^2)$

$= 32x^3-2xy^2$

10) Determine the equation of the parabola. Express your answer in standard form.

$y = a(x+4)(x-8)$

$8 = a(0+4)(0-8)$

$8 = a(4)(-8)$

$\frac{8}{-32} = \frac{a(-32)}{-32}$

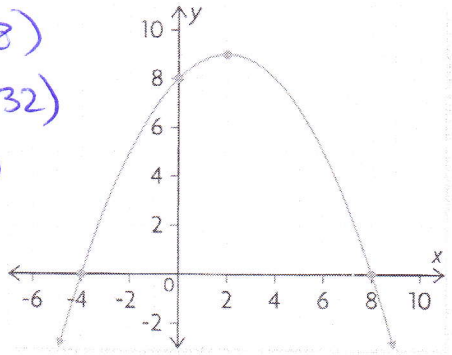
$a = -\frac{1}{4}$

$y = -\frac{1}{4}(x+4)(x-8)$

$= -\frac{1}{4}(x^2-8x+4x-32)$

$= -\frac{1}{4}(x^2-4x-32)$

$= -\frac{1}{4}x^2+x+8$



11) Factor each expression.

a. $20x^2 - 4x$

$= 4x(5x-1)$

b. $3n^2 - 6n + 15$

$= 3(n^2-2n+5)$

c. $-2x^3 + 6x^2 + 4x$

$= -2x(x^2-3x-2)$

d. 6a(3-7a) – 5(3-7a)

$= (6a-5)(3-7a)$

e. $x^2 + 16x + 63$

$= (x+9)(x+7)$

P: 63

S: +9+7 = 16

I: +9 & +7

f. $x^2 + 6x - 27$

$= (x+9)(x-3)$

P: -27

S: +9-3 = 6

I: +9 & -3

g. $x^2 - 7x - 60$

$= (x+5)(x-12)$

P: -60

S: 5-12 = -7

I: 5 & -12

h. $5x^2 - 5x - 100$

$= 5(x^2-x-20)$

$= 5(x-5)(x+4)$

GCF

P: -20

S: -5+4 = -1

I: -5 & 4

12) Examine the relation $y = x^2 + 7x + 12$

a. Write the relation in factored form.

b. Determine the coordinates of the x-intercepts.

c. Determine the coordinates of the vertex.

d. State the minimum value of the relation and where the minimum value occurs.

a) $y = x^2+7x+12$ P: 12 S: 4+3=7 I: 4 & 3

$=(x+4)(x+3)$

b) $=(x+4)(x+3)$

$x+4=0$ OR $x+3=0$

$x=-4$ $x=-3$

Zeroes: $(-4,0)$ & $(-3,0)$

c) Zeroes: -4 & -3

Axis of symmetry $= \frac{-4-3}{2}$

$x = -3.5$

sub $x=-3.5$ into equation

$y = (-3.5)^2+7(-3.5)+12$

$y = -0.25$

Vertex $(-3.5, -0.25)$

d) The minimum value is $y=-0.25$ and it occurs at $x=-3.5$.

13) Explain the strategy you would use to factor each trinomial.

a. $15x^2 - 4x - 4$

P: -60

S: $6-10=-4$

I: 6 & -10

b. $7a^2 + 6a - 16$

P: -112

S: $-8+14=+6$

I: -8 & 14

Example:

$= 7a^2+14a-8a-16$

$= 7a(a+2)-8(a+2)$

$= (7a-8)(a+2)$

c. $20x^2 + 3x - 2$

P: -40

S: $8-5=3$

I: 8 & -5

d. $20y^2 - 17y - 10$

P: 200

S: $8-25=-17$

I: 8 & -25

14) Factor each expression.

a. $7x^2 - 19x - 6$

P: -42

S: $2-21=-19$

I: 2 & -21

$= 7x^2-21x+2x-6$

$= 7x(x-3)+2(x-3)$

$= (7x+2)(x-3)$

b. $4a^2 + 23a + 15$

$= 4a^2+20a+3a+15$

$= 4a(a+5)+3(a+5)$

$= (4a+3)(a+5)$

P: 60

S: $20+3=23$

I: 20 & 3

c. $12a^2 - 16a + 5$

P: 60

S: $-6-10=-16$

I: -6 & -10

$= 12a^2-6a-10a+5$

$= 6a(2a-1)-5(2a-1)$

$= (6a-5)(2a-1)$

d. $6a^2 - 11ay - 10y^2$

$= 6a^2+4ay-15ay-10y^2$

$= 2a(3a+2y)-5y(3a+2y)$

$= (2a-5y)(3a+2y)$

P: -60

S: $4-15=-11$

I: 4 & -15

15) Erica and Asif sell newly designed digital watches. The profit on the watches they sell is determined by the relation $P = -2n^2 + 120n - 1000$, where n is the number of watches sold and P is the profit in dollars.

a. What are the break-even points for Erica and Asif?

$= -2(n^2 - 60n + 500)$

$= -2(n - 10)(n - 50)$

$n - 10 = 0$ OR $n - 50 = 0$

$n = 10$ $\quad$ $n = 50$

P: 500

S: $-10 - 50 = -60$

I: $-10, -50$

∴ the break-even points are 10 & 50 watches.

b. What is the maximum profit that Erica and Asif can earn?

axis of symmetry $= \frac{10+50}{2}$

$x = 30$

sub $x = 30$ into equation

$= -2(30)^2 + 120(30) - 1000$

$= 800$

∴ the maximum profit is $800

16) Factor each expression

a. $144x^2 - 25$

$\sqrt{144}$, $\sqrt{25}$

$= (12x - 5)(12x + 5)$

b. $18x^5 - 512xy^2$

$= 2x(9x^4 - 256y^2)$

$= 2x(3x - 16y)(3x + 16y)$

c. $(x + 5)^2 - y^2$

$= (x + 5 - y)(x + 5 + y)$

d. $7x^2 - 26x - 8$

P: -56

S: $2 - 28 = -26$

I: 2 & -28

$= 7x^2 - 28x + 2x - 8$

$= 7x(x - 4) + 2(x - 4)$

$= (7x + 2)(x - 4)$

e. $18ac - 12a - 15c + 10$

$= 6a(3c - 2) - 5(3c - 2)$

$= (6a - 5)(3c - 2)$

f. $20x^2 + 61x + 45$

P: 900

S: $36 + 25 = 61$

I: 36 & 25

$= 20x^2 + 36x + 25x + 45$

$= 4x(5x + 9) + 5(5x + 9)$

$= (4x + 5)(5x + 9)$

g. $2s^2 + 3s - 5$

P: -10

S: $5 - 2 = 3$

I: 5 & -2

$= 2s^2 - 2s + 5s - 5$

$= 2s(s - 1) + 5(s - 1)$

$= (2s + 5)(s - 1)$

h. $z^4 - 4z^2 - 32$

$= (z^2 + 4)(z^2 - 8)$

i. $9 - 30g + 25g^2$

$= (5g - 3)(5g - 3)$

$= (5g - 3)^2$

17) Determine the coordinates of the vertex of each relation.

a. $y = x^2 - 10x + 24$

$= (x - 4)(x - 6)$ $\quad$ Vertex $(5, -1)$

$x - 4 = 0$ or $x - 6 = 0$

$x = 4$ $\quad$ $x = 6$

axis of symmetry $= \frac{4+6}{2}$

$x = 5$

sub $x = 5$ into equation

$y = (5)^2 - 10(5) + 24$

$= -1$

b. $y = -5x^2 + 500$

$= -5(x^2 - 100)$

$= -5(x - 10)(x + 10)$

$x - 10 = 0$ OR $x + 10 = 0$

$x = 10$ $\quad$ $x = -10$

axis of symmetry $= \frac{10-10}{2}$

$x = 0$

sub $x = 0$ into equation

$y = -5(0)^2 + 500$

$y = 500$

Vertex $(0, 500)$

c. $y = 4x^2 + 16x$

$= 4x(x + 4)$

$4x = 0$ $\quad$ $x + 4 = 0$

$x = 0$ $\quad$ $x = -4$

axis of symmetry $= \frac{-4+0}{2}$

$x = -2$

sub $x = -2$ into equation

$y = 4(-2)^2 + 16(-2)$

$= -16$

Vertex $(-2, -16)$